

TOPICS : Units & Measurement Error Analysis

- If the energy, $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the speed of light, then the values of p , q and r are, respectively.
 - $\frac{-1}{2}, \frac{1}{2}$ and $\frac{5}{2}$
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- The density of a cube is measured by measuring its mass and the length of its side. If the maximum errors in the measurements of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is
 - 7%
 - 5%
 - 9%
 - 3%
- Turpentine oil is flowing through a tube of length l and radius r . The pressure difference between the two ends of the tube is P . The viscosity of oil is given by, $\eta = \frac{P(r^2 - x^2)}{4vl}$, where v is the velocity of oil at a distance x from the axis of the tube. The dimension of η is
 - $[M^0 L^0 T^0]$
 - $[MLT^{-1}]$
 - $[ML^2 T^{-2}]$
 - $[ML^{-1} T^{-1}]$
- The relation between force F and density d is $F = \frac{x}{\sqrt{d}}$. The dimensions of x are
 - $[L^{-1/2} M^{3/2} T^{-2}]$
 - $[L^{-1/2} M^{1/2} T^{-2}]$
 - $[L^{-1/2} M^{3/2} T^{-2}]$
 - $[L^{-1} M^{1/2} T^{-2}]$
- In the formula, $X = 3YZ^2$; X has dimensions of capacitance and Z has dimensions of magnetic induction. The dimensions of Y are
 - $[M^{-3} L^{-2} T^{-2} A^4]$
 - $[ML^{-2} T^2 A^2]$
 - $[M^{-3} L^{-2} T^{-4} A^4]$
 - $[M^{-3} L^{-2} T^8 A^4]$
- If E , m , l and G denote energy, mass, angular momentum and gravitational constant respectively, the quantity (El^2/m^5G^2) has the dimensions of
 - angle
 - length
 - mass
 - time
- Which two of the following five physical parameters have the same dimensions ?
 - Energy density
 - refractive index
 - dielectric constant
 - young's modulus
 - 1 and 4
 - 1 and 5
 - 2 and 4
 - 3 and 5
- In a new system of units, unit of mass is 10 kg, unit of length is 1 km and unit of time is 1 minute. The value of 1 joule in this new hypothetical system is
 - 3.6×10^{-4} new units
 - 6×10^7 new units
 - 10^{11} new units
 - 1.67×10^{-4} new units
- The ratio of the dimensions of Planck constant and that of moment of inertia is the dimensions of
 - time
 - frequency
 - angular momentum
 - velocity
- The dimensions of resistance \times capacitance are same as that of
 - current
 - energy
 - frequency
 - time-period

TOPICS : Units & Measurement Error Analysis (SOLUTION)

1. (a) : $E = G^p h^q c^r$... (i)
Equating dimensions on both sides of equation (i), we get
 $[M^1 L^2 T^{-2}] = [M^{-1} L^3 T^{-2}]^p [ML^2 T^{-1}]^q [LT^{-1}]^r$
 $= [M^{-p+q} L^{3p+2q+r} T^{-2p-q-r}]$
Applying principle of homogeneity of dimensions, we get
 $-p + q = 1$... (ii)
 $3p + 2q + r = 2$... (iii)
 $-2p - q - r = -2$... (iv)
Adding (iii) and (iv), we get
 $p + q = 0$... (v)
Adding (ii) and (v), we get
 $2q = 1$ or $q = \frac{1}{2}$
From (ii), $p = q - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$
Substituting the values of p and q in equation (iii), we get
 $-\frac{3}{2} + 1 + r = 2$ or $r = \frac{5}{2}$

Hence, $p = -\frac{1}{2}, q = \frac{1}{2}, r = \frac{5}{2}$

2. (c) : $\therefore \rho = \frac{M}{L^3}$
 $\therefore \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + 3 \frac{\Delta L}{L} = 3\% + 3(2\%) = 9\%$

3. (d) : Dimensions of $P = [ML^{-1}T^{-2}]$
Dimensions of $r = [L]$
Dimensions of $v = [LT^{-1}]$
Dimensions of $l = [L]$
 \therefore Dimensions of $\eta = \frac{[P](r^2 - x)^2}{[v][l]} = \frac{[ML^{-1}T^{-2}][L^2]}{[LT^{-1}][L]}$
 $= [ML^{-1}T^{-1}]$

4. (a) : Given : $F = \frac{x}{\sqrt{d}}$
 $\therefore x = F\sqrt{d}$
 $[x] = [L^1 M^1 T^{-2}][L^{-3/2} M^1 T^0]^{1/2}$
 $= [L^1 M^1 T^{-2}][L^{-3/2} M^{1/2} T^0] = [L^{-1/2} M^{3/2} T^{-2}]$

5. (d) : As $q = CV$,
 $\Rightarrow C = \frac{q}{V} = \frac{q^2}{W} \left[\text{as } V = \frac{W}{q} \right]$
 $[X] \rightarrow [C] = \left[\frac{A^2 T^2}{ML^2 T^{-2}} \right] = [M^{-1} L^{-2} T^4 A^2]$
 $F = BIl \sin \theta, [B] = \left[\frac{F}{Il} \right]$
 $[Z] \rightarrow [B] = \left[\frac{MLT^{-2}}{AL} \right] = [ML^0 T^{-2} A^{-1}]$
Given: $X = 3YZ^2 \Rightarrow Y = X/(3Z^2)$
or $[Y] = \frac{[X]}{[Z]^2} = \frac{[M^{-1} L^{-2} T^4 A^2]}{[ML^0 T^{-2} A^{-1}]^2} = [M^{-3} L^{-2} T^8 A^4]$

6. (a) : $[E] = [ML^2 T^{-2}], [m] = [M], [l] = [ML^2 T^{-1}]$
 $[G] = [M^{-1} L^3 T^{-2}]$
 $\therefore \left[\frac{El^2}{m^5 G^2} \right] = \frac{[ML^2 T^{-2}] \cdot [M^2 L^4 T^{-2}]}{[M^5] \cdot [M^{-2} L^6 T^{-4}]} = [M^0 L^0 T^0]$

As angle has no dimension, this has the same dimension as the angle.

7. (a) : [Energy density] = $\left[\frac{\text{Work done}}{\text{Volume}} \right]$
 $= \frac{[MLT^{-2}L]}{[L^3]} = [ML^{-1}T^{-2}]$
[Young's modulus] = $[Y] = \left[\frac{\text{Force}}{\text{Area}} \right] \times \frac{[l]}{[\Delta l]}$
 $= \frac{[MLT^{-2}][L]}{[L^2][L]} = [ML^{-1}T^{-2}]$

The dimensions of 1 and 4 are the same.

8. (a) : The dimensional formula of energy is $[ML^2 T^{-2}]$.
 $n_2 = 1 \left[\frac{1 \text{ kg}}{10 \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{1 \text{ km}} \right]^2 \left[\frac{1 \text{ s}}{1 \text{ min}} \right]^{-2}$
 $= \frac{1}{10} \times \frac{1}{10^6} \times \frac{1}{(60)^2} = \frac{3600}{10^7} = 3.6 \times 10^{-4}$

9. (b): Planck's constant, $h = \frac{E}{\nu}$
 $= \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

Moment of inertia, $I = [ML^2]$

$$\frac{h}{I} = \frac{[ML^2T^{-1}]}{[ML^2]} = [T^{-1}] = \text{frequency}$$

10. (d): Resistance = $\frac{\text{voltage}}{\text{current}}$ and

$$\text{Capacitance} = \frac{\text{charge}}{\text{voltage}}$$

$$\therefore \text{Resistance} \times \text{capacitance}$$

$$= \frac{\text{voltage}}{\text{current}} \times \frac{\text{charge}}{\text{voltage}} = \frac{\text{charge}}{\text{current}}$$

$$= \frac{\text{current} \times \text{time period}}{\text{current}} = \text{time period.}$$